

Modal Logic: Tableaux and counter-models

Exercise 1 : Validity in K

Using tableaux, show that the following forms are valid in K:

- a) $(\Box p \wedge \Box q) \vDash_K \Box (p \wedge q)$
- b) $\Diamond p \vee \Diamond q \vDash_K \Diamond (p \vee q)$
- c) $\Box (p \wedge q) \vDash_K \Box p \wedge \Box q$
- d) $\Box p, \Box (p \rightarrow q) \vDash_K \Box p \rightarrow \Box q$
- e) $\Diamond p \wedge \Diamond q \vDash_K \Diamond (p \vee r)$
- f) $\Diamond \neg q \rightarrow (\Box \Box \neg \neg p) \vDash_K \Box \Box p \vee \Box q$

Exercise 2 : Counter-models

Using tableaux, build counter-models in K for the following forms

- a) $\Box p \rightarrow \Box q \not\vDash_K \Box (p \rightarrow q)$
- b) $\Diamond p \wedge \Diamond q \not\vDash_K \Diamond (p \wedge q)$
- c) $\neg \Diamond \neg p \not\vDash_K \Diamond p$
- d) $\Box p, \Box \neg q \rightarrow \Box \neg p \not\vDash_K \Box q$

Exercise 3 : Valid or invalid?

Using tableaux, check whether or not the following forms are valid in K

- a) $\Box (p \vee q) \vDash_K (\Box p \vee \Box q)$
- b) $\emptyset \vDash_K \Box (\neg \Box p \rightarrow (\Diamond \neg p \wedge \Diamond q))$
- c) $\Diamond p \vDash_K \Box (\Diamond p \vee \Diamond q)$
- d) $\Box q, \Diamond p \vee \Diamond q \vDash_K \Diamond (p \wedge q)$
- e) $\Box p \vDash_K \Diamond p$
- f) $\Box p \vDash_K \Box \Box p$
- g) $p \vDash_K \Box \Diamond p$
- h) $\Diamond p \vDash_K \Box \Diamond p$