

Tableaux for first-order logic

January 31, 2025

1

Build tableaux to show that:

1. $\forall x \neg Fx \vdash \exists x (Fx \rightarrow Gx)$
2. $\forall x \neg Fx \vdash \neg \exists x Fx$
3. $\neg \exists x Fx \vdash \forall x \neg Fx$
4. $\exists x \neg Fx \vdash \neg \forall x Fx$
5. $\neg \forall x Fx \vdash \exists x \neg Fx$
6. $(\forall x)(Fx \rightarrow (\forall y)Gy) \vdash (\forall x)(\forall y)(Fx \rightarrow Fy)$

2

Build tableaux to show that the following forms are invalid. Use them to find counter-models.

1. $\exists x Fx, \forall x (Fx \rightarrow Gx) \vdash Ga$
2. $\exists x Fx, Ga \vdash \exists x (Fx \wedge Gx)$

3

Find tableaux rules for the identity symbol. You should describe rules for $T : a = a$, $F : a = a$ and $T : a = b$.

4

In philosophy there is an important difference between the claim that everything has a cause, and the claim that there is a cause of everything. Consider the arguments:

- Everything has a cause. Therefore, there is a cause of everything.
- There is a cause of everything. Therefore, everything has a cause.

Formalize these arguments, and build tableaux to check whether they are valid or not. Discuss.

5

In the context of the discussion about soundness, prove that when a tree is extended with $V : \phi \rightarrow \psi$ or with $F : \phi \rightarrow \psi$, it remains open.