Advanced Logic, 1

Exercise 1

Let L be a language with signature={ $<$ }, and let us consider $\phi = x_1 < x_2$.

a) $M = (D, \bar{<})$, with $D = \{1,3,5\}$, $\bar{<} = \{(1,3),(1,5),(3,5)\}$. Find an assignment σ such that $M \vDash_\sigma \phi$. Find another assignment σ' such that $M \nvDash_{\sigma'} \phi$. Do we have $M \vDash \phi$? Why ?

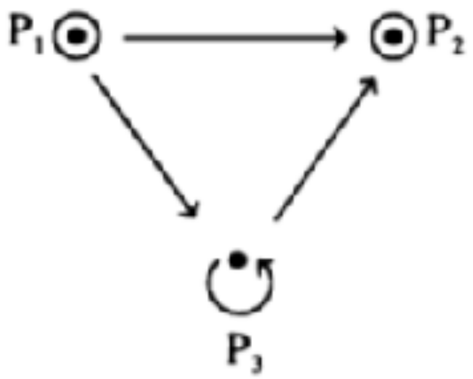
b) $\psi = \exists x(x < y)$. Find an assignment σ such that $M \vDash_\sigma \psi$.

Exercise 2

Consider a language with three constant symbols a, b, c, a unary predicate letter A, and a binary predicate letter R.

a) Give the signature of the language

b) Let us consider the following $\mathscr{L}$-structure $\mathscr{M}$:



P_1, P_2 et P_3 are the interpretations of the three constant symbol; the extension of A is the set of circled points; the points related by an arrow are in the R relation.

Give a precise description of the interpretation $\mathscr{I}$ of this model.

c) Give the truth values of the following formulae in $\mathscr{M}$; justify your answer :

- $\exists x \exists y \exists z (Rxy \wedge Ay \wedge \neg Az)$
- $\forall x Rxx$
- $\forall x (Rxx \leftrightarrow \neg Ax)$
- $\exists x \exists y (Rxy \wedge \neg Ax \wedge \neg Ay)$
- $\forall x (Rxx \rightarrow \exists y (Rxy \wedge Ay))$
- $\forall x (Ax \rightarrow \exists y Rxy)$
- $\exists x \exists y (Rxy \wedge \neg Ryx \wedge \exists z (Rxz \wedge Rzy))$