First-Order Logic: Translation Exercise

Solutions

Advanced Logic for Philosophy & Cognitive Science

Interpretation Key (Reminder)

Constants: a = Anna, b = Beatrice, c = Carlos, d = David
Monadic: Px (x is a philosopher), Lx (x is a logician), Sx (x is a student), Rx (x is rational)
Binary: Axy (x admires y), Txy (x teaches y), Kxy (x knows y)
Triadic: $Ixyz$ (x introduces y to z), $Gxyz$ (x gives y to z)

Solutions with Commentary

1. Anna is a philosopher who admires every logician.

$$Pa \wedge \forall x(Lx \rightarrow Aax)$$

Commentary: A conjunction of a monadic predication and a universally quantified conditional. The relative clause "who admires every logician" is rendered as a universal statement about Anna. Common error: writing $\forall x(Lx \wedge Aax)$, which would mean Anna admires everything and everything is a logician.

2. Someone admires Carlos, but Carlos admires only himself.

$$\exists x(Axc) \wedge \forall x(Acx \rightarrow x = c)$$

Commentary: The key challenge is "only himself." This means: *for any x, if Carlos admires x, then x is Carlos.* This does not entail that Carlos admires himself—it merely restricts his admiration to himself *if* he admires anyone.

Alternative: $\exists x(Axc) \wedge \forall x(x \neq c \rightarrow \neg Acx)$ (equivalent by contraposition)

Note: If we want to assert that Carlos *does* admire himself, we would add Acc as a conjunct.

3. Every philosopher who is not a logician admires some logician.

$$\forall x\big((Px \wedge \neg Lx) \rightarrow \exists y(Ly \wedge Axy)\big)$$

Commentary: The subject is restricted ("philosopher who is not a logician"), yielding a complex antecedent. The quantifier "some" in the consequent takes narrow scope. Common error: giving the existential wide scope, i.e., $\exists y(Ly \wedge \forall x((Px \wedge \neg Lx) \rightarrow Axy))$, which would mean there is a *single* logician admired by all such philosophers.

4. No student knows every philosopher.

$$\neg\exists x\big(Sx \wedge \forall y(Py \to Kxy)\big)$$

Commentary: "No F is G" is standardly rendered as $\neg\exists x(Fx \wedge Gx)$. Here, being G is "knowing every philosopher."

Alternative: $\forall x\big(Sx \to \neg\forall y(Py \to Kxy)\big)$

Alternative: $\forall x\big(Sx \to \exists y(Py \wedge \neg Kxy)\big)$

All three are logically equivalent.

5. **David teaches someone who is rational, and everyone David teaches is a student.**

$$\exists x(Rx \wedge Tdx) \wedge \forall x(Tdx \to Sx)$$

Commentary: A straightforward conjunction. Note that the two conjuncts together entail that David teaches at least one rational student, but this should not be explicitly stated—it follows logically.

6. **Beatrice introduced Anna to everyone she knows except herself.**

$$\forall x\big((Kbx \wedge x \neq b) \to Ibax\big)$$

Commentary: The exception clause "except herself" restricts the domain of the universal quantifier. We interpret "she" as referring to Beatrice (the subject). Thus: for everyone x, if Beatrice knows x and x is not Beatrice, then Beatrice introduced Anna to x.

Note: This does *not* assert that Beatrice knows herself or that she didn't introduce Anna to herself—it is silent on those points.

Alternative: $\forall x(Kbx \to (x \neq b \to Ibax))$ (equivalent)

7. **There is exactly one logician whom every philosopher admires.**

$$\exists x\Big(Lx \wedge \forall y(Py \to Ayx) \wedge \forall z\big((Lz \wedge \forall y(Py \to Ayz)) \to z = x\big)\Big)$$

Commentary: "Exactly one" requires both existence and uniqueness. The formula asserts:

(i) There is a logician x whom every philosopher admires.

(ii) Any logician z whom every philosopher admires is identical to x.

Simpler alternative (if we drop the requirement that the unique individual be a logician in the uniqueness clause):

$$\exists x\Big(Lx \wedge \forall y(Py \to Ayx) \wedge \forall z\big(\forall y(Py \to Ayz) \to z = x\big)\Big)$$

This version says there is exactly one *thing* (who happens to be a logician) that every philosopher admires. The two readings are subtly different; either is acceptable depending on interpretation.

8. If anyone is a philosopher, then Carlos introduced that person to some logician.

$$\forall x\big(Px \to \exists y(Ly \land Icxy)\big)$$

Commentary: "Anyone" here functions as a universal quantifier ("if anyone is F, then ..." $\equiv$ "for all x, if x is F, then ..."). The anaphoric "that person" is bound by the same quantifier. Common error: using an existential for "anyone."

9. Anna gives something to Beatrice that Beatrice gives to no one else.

$$\exists x\Big(Gaxb \land \forall y(y \neq b \to \neg Gbxy)\Big)$$

Commentary: "No one else" is interpreted as "no one other than Beatrice." The sentence says there is some object x such that Anna gives x to Beatrice, and for any person y distinct from Beatrice, Beatrice does not give x to y. (Beatrice may or may not "give it to herself"—an odd locution—but the sentence is silent on this.)

Alternative: $\exists x(Gaxb \land \neg\exists y(y \neq b \land Gbxy))$ (equivalent)

10. Every student who admires a philosopher admires someone who is not identical to themselves.

$$\forall x\Big(\big(Sx \land \exists y(Py \land Axy)\big) \to \exists z(z \neq x \land Axz)\Big)$$

Commentary: The subject is "every student who admires a philosopher"—i.e., students who admire *at least one* philosopher. The predicate is "admires someone who is not identical to themselves," where "themselves" is bound by the subject quantifier. This tests understanding of reflexive pronouns and identity.

Note: The sentence is trivially satisfied by any student who admires a philosopher, since the philosopher they admire (if distinct from themselves) witnesses the existential in the consequent. The sentence rules out only the case where a student admires some philosopher(s) but admires *only* themselves.